① $I = \iint_D \left(\frac{x^2}{9} + \frac{y^2}{16}\right) dx\,dy$

$I = \int_{\theta=0}^{2\pi} \int_{r=0}^{3} r^2 \left(\frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{16}\right) r\,dr\,d\theta$

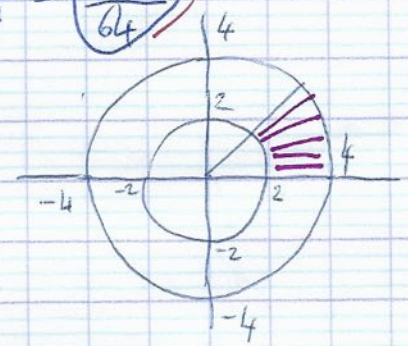
$= \int_{\theta=0}^{2\pi} \left(\frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{16}\right) d\theta \int_{r=0}^{3} r^3\,dr$

$= \int_{\theta=0}^{2\pi} \left(\frac{\cos(2\theta)+1}{2\times 9} + \frac{1-\cos(2\theta)}{2\times 16}\right) d\theta \times \left[\frac{r^4}{4}\right]_0^3$

$= \left\{\frac{1}{2}\left[\frac{\sin 2\theta}{18} + \frac{\cos 2\theta}{32}\right]_0^{2\pi} + \frac{2\pi}{2}\left(\frac{1}{9}+\frac{1}{16}\right)\right\} \times \frac{34}{4}$

$= \frac{2\pi}{2} \times \frac{25}{144} \times \frac{81}{4} = \frac{225\pi}{64} \approx 11{,}045$

② $\iint_D \frac{dx\,dy}{x^2+xy+y^2}$



$I = \iint \frac{r\,dr\,d\theta}{r^2 + r^2\cos\theta\sin\theta}$

$= \iint \frac{1}{r} \frac{dr\,d\theta}{1+\cos\theta\sin\theta} = \underbrace{\int_{\theta=0}^{\pi/4} \frac{d\theta}{1+\cos\theta\sin\theta}}_{I_\theta} \times \underbrace{\int_{r=2}^{4} \frac{dr}{r}}_{I_r}$

bioche $\theta+\pi$

$I_r = \ln 2$

$I_\theta = \int_0^{\pi/4} \frac{d\theta/\cos^2\theta}{\frac{1}{\cos^2\theta} + \tan\theta} = \int_0^{\cdot} \frac{dt}{1+t^2+t}$ avec $t = \tan\theta$

$\int \frac{dt}{1+t+t^2} = \frac{2}{\sqrt{3}} \operatorname{atan}\left(\frac{2t+1}{\sqrt{3}}\right)$

$I_\theta = \left[\frac{2}{\sqrt{3}} \arctan\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^1$

$I_\theta = \frac{2}{\sqrt{3}}\left(\arctan\sqrt{3} - \arctan\frac{1}{\sqrt{3}}\right)$

$I_\theta = \frac{2}{\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{3\sqrt{3}}$

$I = \frac{\pi \ln 2}{3\sqrt{3}}$